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Thermal action of a short light pulse on biological tissues

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Abstract

A simple model for optical and thermal properties of two-component biological tissues is proposed as applied to studies of thermal fields under external illumination. The model comprises a small number of varying input parameters to enable one to find all the optical characteristics required to compute light fields in tissue and to state the thermal source function. Thermal parameters of tissues determining heat transfer in a two-component medium are calculated with accounting for heat exchange conditions between the components and at the interface with various external media. A set of heat conduction equations is stated for the two-component medium simulating biological tissues. Its analytical solution is derived. Spatial distributions of the fluence rate and temperature over the tissue depth are investigated at varying time moments after the irradiation by a short light pulse. Localized absorption of light by blood vessels and its effect on optical parameters of the medium, more intense heating of blood as compared with its surrounding (basic) tissue and heat exchange between the blood and tissue, as well as heat transfer at the interface with different environments are taken into account. The solutions are derived via characteristic times of thermal processes to enable one to easy and vividly evaluate the features in tissue heating as well as the effects of optical and thermal parameters on temperature distributions of the components. The calculations are illustrated by examples.

1. Introduction

A problem on biological tissue heating by external illumination is scientifically and practically interesting for various fields of medicine, physiology, laser physics and engineering and their applications, for protection from extreme light exposures, etc. To this end, it is important to have an analytical engineering instrument to estimate temperature regime in a medium enabling one to study general relations of spatial and temporal heat distributions and effects of characteristic parameters on light and thermal fields. It is just the objective of this paper. The problem of light thermal action on tissues is reduced to the construction of a model for optical and thermal characteristics of the medium on the base of the tissue structure and to the solution of radiation and heat transfer equations with using the said characteristics as inputs to evaluate light and thermal fields.

2. Structural model of biological tissues

Let tissue be a two-component medium composing of the tissue itself (bloodless basic tissue) and blood vessels randomly distributed over it (all the capillary orientations are equally probable and blood velocities are the same). Assume that the medium is macroscopically uniform in the optical and thermal senses and its properties do not depend on spatial coordinates. This simplifying assumption is made to do not complicate the problem by introducing a lot of additional parameters that, on the one hand, are known only approximately in many cases to be estimated from rather rough considerations and, on the other hand, often have small effects on light and thermal regimes, but make it essentially difficult to vividly present final results. Suppose further that blood vessels (capillaries) have constant diameter $d = 2.5$–10 μm and length l of a capillary to its bend is

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Nomenclature

A	coefficient from Eqs. (22) and (23)
c	specific heat
C_V	volume fraction of blood vessels
d	mean diameter of a capillary
$\mathrm{erfc}(x)$	complementary error function, $1-(2/\sqrt{\pi})\int_0^x \exp(-y^2)\,dy$
E	fluence rate
f	volume fraction of hemoglobin in erythrocytes
F	auxiliary function from Eqs. (22) and (23)
F_1	auxiliary function from Eqs. (22) and (23)
F_2	auxiliary function from Eq (33)
G	hematocrit
Gr	Grashoff number
h	heat transfer parameter at the interface between tissue and its environment, H/κ
H	heat transfer coefficient at the interface between tissue and its environment
I_p	modified Bessel function of p order
k	absorption coefficient, cm^{-1}
l	mean length of a capillary
Nu	Nusselt number
OD	blood oxygenation degree
Pe	Peclet number
Pr	Prandtl criterion
r_1	light reflection coefficient of tissue surface
r_2	light diffuse reflection coefficient
R	light reflection coefficient of tissue layer
Re	Reynolds number
Q	heat quantity
s	effective scattering coefficient, $s'(1-\bar{\mu})$
s'	scattering coefficient
S	thermal heat source function due to external illumination
t	time after illumination
T	temperature
U	thermal heat source function due to human organism action
w	characteristic size of light spot at the entrance to medium
W	auxiliary function from Eqs. (8) and (9)
X	characteristic size of a problem
Y	auxiliary function from Eq. (17)
z	depth
z_0	light penetration depth

Greek symbols

α	parameter of heat exchange between blood and basic tissue
α_1	factor of heat exchange between blood and basic tissue, $\alpha/[C_V(1-C_V)]$
β	depth extinction coefficient, $\sqrt{3k\varepsilon}$
δ	delta pulse
ε	effective extinction coefficient, cm^{-1}, $\varepsilon'(1-g)$
ε'	extinction coefficient, $k+s'$
φ	light incidence angle
κ	thermal conductivity
λ	light wavelength, nm
$\bar{\mu}$	mean cosine of phase function
ν	kinematic viscosity
η	thermal diffusivity, $\kappa/(c\rho)$
ρ	density
τ	characteristic time of a thermal process
χ	effective absorption diameter of a capillary, $k_2\pi d/4$

Subscripts

0	initial time moment
1	basic tissue
2	blood
3	environmental medium
b	background
c	convection
h	heat transfer to environment
r	radiation
w	radial heat transfer
α	heat exchange between blood and basic tissue
η	depth heat transfer

Superscripts

max	maximal
$*$	"sieve" effect

much larger than d. Such a vessel element can be represented by a long cylinder. Besides, let tissue properties be independent of light energy, i.e. external irradiation does not lead to any physical, chemical, phase or other changes in the tissue. So the varying structural parameters of the model are d and vessel volume content C_V equal to a volume fraction of the medium occupied by the vessels. The values of C_V range from 0.01 to 0.1 to cover roughly the properties of normal and pathological tissues.

3. Optical characteristics of tissues

Light transfer through a scattering and absorbing medium is determined by its structural or geometrical parameters and by three phenomenological characteristics, namely by absorption k and scattering s' coefficients and by phase function of a unit volume [1].

Due to high turbidity of tissues, one often uses the asymptotic approximation of the radiative transfer theory [2] comprising not three, but two optical parameters,

namely k and effective extinction coefficient $\varepsilon = \varepsilon'(1-\bar{\mu})$, where $\varepsilon' = k + s'$, $\bar{\mu}$ is the mean cosine of phase function. According to the above structural model, one needs to know these two characteristics individually for basic tissue and blood. Then the resulting coefficients can be calculated by formulas (here and below subscripts 1 and 2 refer to tissue and blood, respectively):

$$k = (1 - C_V)k_1 + C_V k_2, \tag{1}$$

$$\varepsilon = \frac{(1 - C_V)\varepsilon'_1 + C_V\varepsilon'_2}{(1 - C_V)s'_1 + C_V s'_2}[(1 - C_V)s_1 + C_V s_2], \tag{2}$$

where $s_{1,2} = s'_{1,2}(1 - \bar{\mu}_{1,2})$, or under the assumption of $k_{1,2} \ll s'_{1,2}$

$$\varepsilon = (1 - C_V)\varepsilon_1 + C_V\varepsilon_2. \tag{3}$$

Let the said two-parameter spectral model $k(\lambda)$ and $\varepsilon(\lambda)$ be constructed. Publications usually give data on k and ε values typical for different tissues with varying vessel volume contents. To solve the optical problem (to evaluate light field), these data are sufficient. However, the thermal regime of individual tissue components depends highly on C_V owing to their different absorption. Therefore, the model should include spectra $k_1(\lambda)$, $k_2(\lambda)$ and vessel content as a varying parameter. We believe that it is most convenient for the problem studied to use model [3] based on thoroughly calibrated measurements and comparisons with other experiments. This model represents k_1 as a sum of background absorption k_{1b} and that of melanin k_{1m} as applied to skin. At this stage, we neglect by k_{1m} values. The following approximation for k_{1b} is proposed

$$k_{1b}\ [\mathrm{cm}^{-1}] = 0.244 + 85.4\exp\left[-\frac{\lambda\ [\mathrm{nm}] - 154}{66.2}\right]. \tag{4}$$

Effective scattering coefficient s_1 was computed [3] by relations similar to the Mie formulas, which gave scattering characteristics $s_{1\mathrm{Mie}}$ of long cylinders simulating tissue fibers and Rayleigh coefficients $s_{1\mathrm{R}}$ of small (with diameter of about 100 nm) spherical inhomogeneities. The coefficients were approximated by the formulas:

$$s_{1\mathrm{Mie}}\ [\mathrm{cm}^{-1}] = 2 \times 10^5 \lambda\ [\mathrm{nm}]^{-1.5},$$
$$s_{1\mathrm{R}}\ [\mathrm{cm}^{-1}] = 2 \times 10^{12} \lambda\ [\mathrm{nm}]^{-4}, \quad s_1 = s_{1\mathrm{Mie}} + s_{1\mathrm{R}}. \tag{5}$$

Spectra $k_1(\lambda)$ and $\varepsilon_1(\lambda)$ calculated by Eqs. (4) and (5) are given in Table 1 (columns 2 and 3).

Optical properties of blood are well experimentally studied in the literature. The main absorbing components in spectral range 400–850 nm are oxy-HbO_2 and deoxyhemoglobin Hb. Normal blood contains other hemoglobin forms (glycated, carboxy-, methemoglobin) in small concentrations, so their absorption can be neglected. By setting hematocrit G (volume content of erythrocytes in blood), volume fraction f of hemoglobin in erythrocytes, and blood oxygenation degree OD, one can construct spectra $k_2 = Gf[\mathrm{OD}\,k_{\mathrm{HbO_2}} + (1 - \mathrm{OD})k_{\mathrm{Hb}}]$ on the base of known absorption of HbO_2 and Hb($k_{\mathrm{HbO_2}}$ and k_{Hb}). We use data [4,5] recalculated to $G = 0.4$, $f = 0.25$, and $\mathrm{OD} = 0.97$ as such model spectral dependencies. Column 4 of Table 1 gives the absorption coefficients. Effective spectral extinction coefficient $\varepsilon_2(\lambda)$ (column 5) as a model one was taken from [4,5]. The comparison of the data of Table 1 with published ones [3,6–9] shows their good agreement. So the data of Table 1 will be used below to compute light and thermal fields.

From a physical viewpoint, the additive sum of the absorption coefficients of Eq. (1) means uniform averaging of the absorbing components over the whole unit volume of the medium. Meanwhile, highly absorbing blood has small specific volume (small volume content) as pointed out above. This suggests an idea that there can occur a "sieve" (or "non-absorbing holes") effect [10–12], when weakly absorbing portions will transmit a lot of light to provide a larger total transmission than it follows from the uniform absorptivity of a unit volume. In this case, the absorption coefficient [13]

$$k_2^* = C_V k_2[1 - \exp(-\chi)]/\chi, \tag{6}$$

Table 1
Spectral absorption k [cm^{-1}] and effective extinction ε [cm^{-1}] coefficients of basic tissue and blood according to the proposed model

λ, nm	Tissue		Blood	
	k_1	ε_1	k_2	ε_2
400	2.32	105	857	10
450	1.22	70.9	286	7.85
500	0.7	50.6	100	7.43
550	0.46	37.8	200	8.57
600	0.345	29.4	12	8.93
650	0.29	23.6	2.07	9
700	0.266	19.4	1.43	8.57
750	0.254	16.3	1.7	8.1
800	0.249	14	3.15	7.6
850	0.248	12.1	3.9	7.27

where $\chi = k_2\pi d/4$. When basic tissue surrounding vessels absorb light too, the resulting absorption coefficient is $k^* = k_1^* + k_2^*$, where $k_1^* = (1 - C_V)k_1$ is the absorption coefficient of the surrounding medium. If one assumes that light scattering takes place on inhomogeneities of the surrounding medium only, then the scattering coefficient of a unit volume is $s^* = s_1^* = s_1'$, where s_1^* and s_1' are the scattering coefficient of a unit volume and average scattering coefficient, respectively. Using k_2 values from Table 1, one gets factors $[1 - \exp(-\chi)]/\chi = 0.728$, 0.962, 0.995, and 1 at $d = 10$ μm for $\lambda = 400$, 500, 600, and 700 nm, respectively, and 0.967; 0.996; 1, and 1 at $d = 1$ μm. So this effect is essential in the blue spectral range for rather large capillaries, when one should use Eq. (6) to calculate k_2 in Eq. (1) rather than take its values from, for example, a table of the optical characteristics.

4. Thermal characteristics of tissues

The characteristics include specific heat c, density ρ, thermal conductivity κ or thermal diffusivity $\eta = \kappa/(c\rho)$ of the medium, parameters of heat exchange between individual tissue components α and at the interface h with an environmental medium. Each of these quantities depends on many factors, such as tissue kind, physiological conditions of an organism, parameters of the environment, etc. So these dependencies are not clearly defined and non-analytical. This paper has the objective to derive simple and physically substantiated estimations of the said characteristics from general statements of the thermal physics, to compare them with published data, and then to introduce the estimations into the analytical scheme of studying the thermal fields.

We consider here biological tissues with high fluid content (75–90%) having the properties close to water [14]. So let below $c\rho = 4.2 \times 10^6$ J/(m^3 K), $\kappa = 0.6$ J/(m K s), $\eta = 1.4 \times 10^{-7}$ m^2/s as the corresponding characteristics of water without the subscript typical of the component. We do not know the theoretical estimations of the h and α parameters derived from the general viewpoints of the thermal physics as applied to biological tissues. These quantities will be investigated below.

Heat exchange in an organism occurs due to the thermal conductivity and blood motion along vessels. As noted above, the assumption on the chaotic distribution of capillaries has been accepted. Under the external irradiation, blood temperature is different at different points. At the first glance, it could seem that there is an additional source of heat transfer due to the random blood motion over tissue similarly to the thermal conductivity. However, from the general expression of the thermal balance or from the usual heat transfer equation at no directed velocity component, it follows that such an additional source is equal to zero.

Consider first the system of blood vessels—basic tissue. The conception of parameter α follows from the approximate expression for heat quantity Q transferred by blood according to the Newton law [15] for convective heat losses:

$$Q = c\rho\alpha(\overline{T}_2 - \overline{T}_1), \tag{7}$$

where $\overline{T}_1$ and $\overline{T}_2$ are the temperature values of the basic tissue and blood averaged over a unit volume. Compare the heat transfer from vessels to basic tissue due to the convection and thermal conductivity. For the estimations, consider the heat exchange between moving heated blood (water) in a cylindrical vessel and colder external medium (water too). The influence of the walls will be neglected. To this end, calculate the Peclet number $Pe = RePr$, where $Re = v_2X/v$, the Reynolds number; v_2, the perfusion rate; $X = d$, the characteristic size of the problem; v, the kinematic viscosity of the moving medium (blood in our case); $Pr = v/\eta$, the Prandtl criterion. As known [15,16], the molecular thermal conductivity dominates at $Pe \ll 1$ and the convection does at the opposite inequality. If $v_2 = 10^{-3}$ m/s and $d = 5 \times 10^{-6}$ m (typical values for blood), one gets $Pe = 3.5 \times 10^{-2} \ll 1$. Therefore, the convection mechanism can be neglected under heat exchange between capillary blood and its surrounding tissue.

Heat exchange under the action of the molecular thermal conductivity only can be described more strictly than Eq. (7) to therefore estimate α. We will start from a set of heat transfer equations for heated cylindrical vessels and colder surrounding tissue. Then the solution [17] for average temperatures has the form

$$\overline{T}_1 = \widetilde{T}_{10} + C_V(\widetilde{T}_{20} - \widetilde{T}_{10})W(t), \tag{8}$$

$$\overline{T}_2 = \widetilde{T}_{20} - (1 - C_V)(\widetilde{T}_{20} - \widetilde{T}_{10})W(t), \tag{9}$$

where $\widetilde{T}_{10}$ and $\widetilde{T}_{20}$ are the initial tissue and blood temperatures due to, for example, irradiation by a δ light pulse

$$W(t) = \exp(-t_0/t)[I_0(t_0/t) + I_1(t_0/t)], \tag{10}$$

$t_0 = d^2/(8\eta)$, I_0 and I_1 are the modified Bessel functions of the zero and first order, respectively.

If the heat exchange is described via Q of Eq. (7), then one has

$$\overline{T}_1 = \widetilde{T}_{10} + C_V(\widetilde{T}_{20} - \widetilde{T}_{10})[1 - \exp(-\alpha_1 t)], \tag{11}$$

$$\overline{T}_2 = \widetilde{T}_{20} - (1 - C_V)(\widetilde{T}_{20} - \widetilde{T}_{10})[1 - \exp(-\alpha_1 t)], \tag{12}$$

where $\alpha_1 = \alpha/[C_V(1 - C_V)]$. By comparing Eqs. (11) and (12) with Eqs. (8) and (9), it is seen that in Eqs. (11) and (12), approximately accounting the heat exchange, the exact function $W(t)$ is replaced by its estimation $1 - \exp(-\alpha_1 t)$. Compare them. One can see from Eqs. (9) and (12) that functions $1 - W(t)$ and $\exp(-\alpha_1 t)$ are

equal to the temporal changes in the relative temperature of capillaries $T_2' = (T_2 - \widetilde{T}_{20} - T_{ef0})/[(1 - C_V) \times (\widetilde{T}_{20} - \widetilde{T}_{10})]$, where $T_{ef0} = (1 - C_V)\widetilde{T}_{10} + C_V\widetilde{T}_{20}$ is the mean temperature of a unit volume at the initial time moment. The value of $\alpha_1 t_0$ can be fitted so that the both functions are the same for the relative temperature drop by $e = 2.7$ times. It turns out that in this case $t/t_0 = 0.82 = 1/(\alpha_1 t_0)$. It further follows that

$$\alpha_1 = 9.7\eta/d^2 \tag{13}$$

or

$$\alpha = 9.7\eta C_V(1 - C_V)/d^2. \tag{14}$$

The use of Eq. (14) to estimate the heat exchange between the basic tissue and vessels appropriately describes the changes in $\overline{T}_2$ (and $\overline{T}_1$) up to $t/t_0 \approx 0.15$, when the vessel temperature decreases to 17% of its maximal value at $t = 0$ [17]. For $t/t_0 > 2$, when the temperature equalization is essentially completed, approximate Eq. (12) gives more rapid decrease in $\overline{T}_2$. Therefore, one can practically use both Eqs. (11) and (12) and Eqs. (8) and (9).

Consider now parameter $h = H/\kappa$, where H is the heat transfer coefficient and κ is the tissue thermal conductivity. It can be represented as $h = h_c + h_r$, where the first term approximately treats the common action of the heat conductivity and convection and the second one does the radiative heat exchange at moderate differences between $\overline{T}_1$, $\overline{T}_2$, and temperature T_3 of an external medium [18]. Estimate below the effectiveness of the said mechanisms for two cases, namely when tissue is heated in air or water. As before, calculate the Peclet number $Pe = v_3 X/\eta_3$, where v_3 is the velocity of the moving external medium due to the free convection, η_3 is its thermal diffusivity, X is the diameter of the irradiating light spot (characteristic size of the problem for a flat surface). By using [16] and thermal parameters of the two external media [19,20], one can show that the maximal velocity of a medium surrounding a flat surface is $v_3 = 0.1(X\Delta T)^{0.5}$ and $0.043(X\Delta T)^{0.5}$ for air and water, respectively, where X [m] and ΔT [K] is the temperature difference between the tissue surface and external medium. Assuming $X = 5 \times 10^{-3}$ m and $\Delta T = 10$ K gives $Pe \approx 5.5$ and 350 for air and water, respectively. This tells that one should essentially regard only convective motion under the heat exchange of a flat surface.

Heat transfer parameter h_c depends on several dimensionless similarity criteria or numbers, and its specific functional form is determined by a flow type (laminar or turbulent) near a heated surface. A simple criterion for laminar flow is used in [21], namely $GrPr < 10^7$. The Grashoff number [16,21] is $Gr = g\gamma_3 \Delta T X^3/v_3^2$, where g is the gravitational acceleration, γ_3 is the volume expansion coefficient of the external medium, and v_3 is its kinematic viscosity. This criterion is easily seen to be valid in the problem under consideration for typical values of ΔT and X.

Now estimate directly the heat transfer coefficients. As shown in [16,21], there occurs the following relation between the Nusselt Nu and Grashoff numbers during free convection and laminar flow near a vertical heated surface

$$Nu = \frac{HX}{\kappa_3} = [0.508\,Pr^{0.5}(0.952 + Pr)^{-0.25}]\,Gr^{0.25}, \tag{15}$$

where κ_3 is the thermal conductivity of a medium contacting with tissue. By using Eq. (15), one gets

$$h_c = \frac{H}{\kappa} = \frac{\kappa_3}{\kappa X}[0.508\,Pr^{0.5}(0.952 + Pr)^{-0.25}]\,Gr^{0.25}. \tag{16}$$

Let $X = 5 \times 10^{-3}$ m. Then for air environment at $\Delta T = 10$ and 100 K, $h_c = 11.5$ and 20.5 m^{-1}, respectively. For water, h_c values increase by about 180 times. When the surface is horizontal, the estimations give approximately the same h_c values [21], if the heated side is up, and twofold lower, if it is down.

For radiative heat exchange [22], $h_r = 6.2D\Delta T/\kappa$ [1/m], where D is the blackness, to give h_r values of the same order as h_c for air. For water, $h_r \ll h_c$ and one can neglect by h_r in this case.

The above estimations of the heat transfer parameter agree with various literature data [22–24].

By summarizing, one can say that the range of parameter h (in m^{-1}) is 5–50 for environmental air and 10^3–10^4 for water. Under power water flows, $h = 10^4$–10^5 m^{-1}. This should be taken into account while computing the kinetics of thermal processes.

5. Light propagation through tissues

Light propagation through biological tissues was studied in a lot of publications. The whole issues of Applied Optics [25–27] and JOSA [28] were devoted to different aspects of the problem. One usually uses the asymptotic theory [2], two-flux [2,29,30] and diffusion approximations [2,30–32], and their various modifications. The review of the models to describe light fields and light absorption by biological tissues as applied to the problem of their heating under external irradiation was given in [30]. Numerical computations often published in the references for rather simple situations are not justified always, because there are reliable and substantially accurate analytical approaches, which will be used in this paper.

Tissue is a highly turbid medium. The light penetration depth is rather small, so that the width of an irradiating light spot can be regarded as being much larger than the illuminated depth of the medium. From the mathematical viewpoint, this corresponds to the irradiation of an optically thick medium by an infinitely wide

light spot, when the light field structure in the transverse spot section remains constant. The light will be assumed as monochromatic one. To investigate light field in a tissue, we use the well-defined asymptotic approximation of the radiative transfer theory [2].

It is shown in [33,34] that the fluence rate at depth z is the following under illumination of an infinitely thick plane-parallel turbid layer

$$E(z) = E_0(1 - r_1)(1 + R)\exp(-\beta z). \quad (17)$$

Here r_1 is the reflection coefficient of the tissue surface, $R = \exp(-Y)$ is the reflection coefficient of the tissue depth, and $\beta = \sqrt{3k\varepsilon}$ is the depth extinction coefficient. Under directed illumination with incidence angle φ by the surface normal, $Y = 4\sqrt{k/(3\varepsilon)} \times [(3/7)(1 + 2\cos\varphi) + r_2/(1 - r_2)]$, and under diffuse illumination $Y = 4\sqrt{k/(3\varepsilon)}/(1 - r_2)$, where r_2 is the diffuse reflectance at the interface with an external medium for light incidence from the tissue interior. If the external medium is air, one gets $Y = 9.3\sqrt{k/(3\varepsilon)}$ at $r_2 = 0.51$ (for tissue refractive index 1.33) and $\varphi = 0$, and $Y = 8.1\sqrt{k/(3\varepsilon)}$ for the diffuse illumination. When the external medium is water ($r_2 = 0$), one has $Y = 5.1\sqrt{k/(3\varepsilon)}$ and $Y = 4\sqrt{k/(3\varepsilon)}$ correspondingly to the above two cases. Under the illumination by a δ-pulse, E_0 of Eq. (17) is the surface energy density of the pulse at the entrance into the medium.

We will not study the structure of light fields according to Eq. (17) in detail. Note only that the main parameters are reflection coefficient R and depth extinction coefficient. The latter characterizes the depth attenuation rate of the fluence rate. The value of $z_0 = 1/\beta$ is the depth, where the fluence rate decreases by $e = 2.7$ times. Table 2 gives the values of R (for tissue in air at $\varphi = 0$), β, and z_0 for blood volume contents $C_V = 0.01$ and 0.1. One can see the well-known result [6] that the reflection coefficient of biological tissues increases in the long-wavelength spectral region. The higher the blood volume content, the lower the reflectance at any wavelength considered. Penetration depth z_0 changes from fractions to some mm with wavelength increasing and C_V decreasing.

The approach to the "sieve" effect leading to decreasing absorptivity of tissues for a number of cases and, as a result, to increasing reflectivity and transmittivity of a turbid layer was schematically described above. Table 2 estimates this effect as applied to characteristics β, z_0, and R at $C_V = 0.01$ and 0.1 and large enough capillary diameter $d = 40$ μm. The model values of k_1, k_2, and ε were used. The values of β, z_0, and R correspond to the calculation of the absorption coefficient by Eq. (1), those of β^*, z_0^*, and R^* do with accounting for Eq. (6). Table 2 shows that the "sieve" effect even for large capillaries occurs only in the short-wavelength spectral region ($\lambda = 400$–450 nm). Decreasing d makes the effect negligible.

6. Statement of the heat transfer equations and their solution

Various numerical procedures are developed to solve the heat transfer equation for biological tissues. The mains of them are the finite-difference [35–37] and finite-element [38,39] schemes and the Green function method [22,40,41]. These methods provide very ample facilities for the investigations to enable one to compute thermal fields for complex tissue geometry including multi-layer structure [30,40,42], presence of

Table 2
Optical characteristics of tissue and maximal temperatures of blood

λ, nm	k, cm^{-1}	β, mm^{-1}	z_0, mm	R, %	k^*, cm^{-1}	β^*, mm^{-1}	z_0^*, mm	R^*, %	$\Delta T_2^{\max}$, K	$\Delta T_2^{*\max}$, K
$C_V = 0.01$										
400	10.9	5.85	0.171	17.9	5.26	4.07	0.246	30.2	47.2	52.1
450	4.07	2.94	0.34	27.8	3.1	2.57	0.39	32.7	17.1	17.7
500	1.69	1.6	0.624	37.6	1.55	1.53	0.652	39.2	6.43	6.5
550	2.46	1.67	0.599	25.4	1.94	1.48	0.674	29.8	11.8	12.1
600	0.462	0.638	1.57	51.2	0.459	0.636	1.57	51.2	0.847	0.848
700	0.278	0.402	2.49	52.7	0.278	0.402	2.49	52.7	0.102	0.102
800	0.278	0.342	2.93	47.1	0.278	0.342	2.93	47.1	0.216	0.216
$C_V = 0.1$										
400	87.8	16.6	0.06	0.75	31.8	10	0.1	5.3	40.3	42.1
450	29.7	7.95	0.126	3.1	20	6.52	0.153	5.8	13.8	14.1
500	10.6	4.02	0.249	8.6	9.2	3.74	0.267	10.2	5.07	5.15
550	20.4	4.81	0.208	2	15.3	4.16	0.24	3.3	9.52	9.67
600	1.51	1.15	0.866	29.7	1.49	1.15	0.873	30	0.727	0.729
700	0.382	0.472	2.12	47.2	0.382	0.472	2.12	47.2	0.098	0.098
800	0.539	0.476	2.1	35	0.538	0.475	2.11	35.1	0.199	0.199

localized blood vessels [42–44], injury of tissue by radiation [30], water evaporation, changes in chemical reaction rates, and other phase transformations in tissues [30,43], radial heat spreading under irradiation by a narrow light beam [30,36,40,43], etc. Besides the said theoretical studies, there were performed a lot of experiments [45–53] directed, first of all, to the provisions of medical applications. One should note another area of problems related with the investigations of internal temperature fields in blood vessels [42,54]. In spite of the highly developed mathematical or, rather, computer means to solve different problems on heat transfer through biological tissues, numerical methods often mask individual effects of the parameters defining light and heat propagation and, finally, spatial–temporal distributions of the temperature. Accounting for multiple parameters and high variability of the problems on studying the thermal regime, it is highly desirable to have at hands an engineering instrument permitting one to do not "stick" in mathematical cumbersomenesses to get physically transparent and practically valuable and visible results. Such an instrument would also enable one to a prior expose essential and inessential parameters to thereby create prerequisites for simplifying the applications of the known numerical procedures while studying delicate effects. It is just the objective of solving the heat transfer equations in this paper.

Starting from the energy conservation law, one can write down the following set of heat transfer equations for a plane layer of a two-component medium:

$$c\rho(1-C_{\mathrm{V}})\frac{\partial\overline{T}_1}{\partial t}=\kappa(1-C_{\mathrm{V}})\frac{\partial^2\overline{T}_1}{\partial z^2}+Q(\overline{T}_1,\overline{T}_2)+S_1(1-C_{\mathrm{V}})+U(1-C_{\mathrm{V}}), \tag{18}$$

$$c\rho C_{\mathrm{V}}\frac{\partial\overline{T}_2}{\partial t}=\kappa C_{\mathrm{V}}\frac{\partial^2\overline{T}_2}{\partial z^2}-Q(\overline{T}_1,\overline{T}_2)+S_2C_{\mathrm{V}}+UC_{\mathrm{V}}. \tag{19}$$

Here Q is the heat quantity transferred from blood to its surrounding tissue per unit volume pet unit time, S and U are the source functions due to the external irradiation and thermal action of an organism, respectively. Under illumination by a δ-pulse, $S_{1,2}(z,t)=k_{1,2}E(z)\delta(t)$.

As while considering light propagation through the medium, Eqs. (18) and (19) do not treat heat outflow to the dark subsurface regions due to the thermal conductivity. In other words, the above set gives the upper-bound estimation of the temperature rise under external irradiation.

A mechanism of organism heat action U providing temperature $T_{10}=T_{20}\approx 36.6$ °C for a normal-state person under overheating or overcooling is poor studied from a formal mathematical viewpoint. Source U was numerically simulated in [23]. Obviously, the number of input parameters of the model increased very sharply. So assume initially that $U=0$.Set the following initial

$$\overline{T}_1(t=0,z)=\overline{T}_2(t=0,z)=T_{10} \tag{20}$$

and boundary conditions of tissue heat exchange with an external medium by the Newton law [22,38]

$$\frac{\partial\overline{T}_{1,2}(t,z=0)}{\partial z}=h[\overline{T}_{1,2}(t,z=0)-T_3], \tag{21}$$

where T_3 is the temperature of the environment. By using the Laplace transform method [18], one gets the solution to Eqs. (18) and (19) under the conditions of Eqs. (20) and (21)

$$\overline{T}_1(t,z)=T_{10}+\frac{A_1F(t,z)}{2}-A_1F_1(t,z,\beta)\frac{\sqrt{\tau_\eta}}{\sqrt{\tau_\eta}-\sqrt{\tau_{\mathrm{h}}}}+\delta T, \tag{22}$$

$$\overline{T}_2(t,z)=T_{20}+\frac{A_2F(t,z)}{2}-A_2F_1(t,z,\beta)\frac{\sqrt{\tau_\eta}}{\sqrt{\tau_\eta}-\sqrt{\tau_{\mathrm{h}}}}+\delta T. \tag{23}$$

Here

$$\begin{aligned}A_1&=A_{10}\exp(-t/\tau_\alpha)+A_0[1-\exp(-t/\tau_\alpha)],\\ A_2&=A_{20}\exp(-t/\tau_\alpha)+A_0[1-\exp(-t/\tau_\alpha)]\end{aligned} \tag{24}$$

or

$$\begin{aligned}A_1&=A_{10}(1-W)+A_0W,\\ A_2&=A_{20}(1-W)+A_0W,\end{aligned} \tag{25}$$

if the average temperatures over a unit volume are described by Eqs. (11) and (12) or (8) and (9), respectively. Coefficients $A_{10,20}=k_{1,2}E_0(1-r_1)(1+R)/(c\rho)$ give the temperatures of a subsurface unit volume at the initial time moment as if the layer would consist of the basic tissue or blood vessels only. Quantity $A_0=(1-C_{\mathrm{V}})A_{10}+C_{\mathrm{V}}A_{20}$ is the temperature of a unit volume of the layer at $z=0$.

$$\delta T=(T_3-T_{10})F_1(t,z,\beta=0), \tag{26}$$

$$F(t,z)=\exp\left(\frac{t}{\tau_\eta}\right)\left[\exp(-\beta z)\operatorname{erfc}\left(\sqrt{\frac{t}{\tau_\eta}}-\frac{z}{2\sqrt{t\eta}}\right)+\exp(\beta z)\operatorname{erfc}\left(\sqrt{\frac{t}{\tau_\eta}}+\frac{z}{2\sqrt{t\eta}}\right)\right], \tag{27}$$

$$F_1(t,z,\beta)=\exp\left(\frac{t}{\tau_\eta}\right)\left[\exp(\beta z)\operatorname{erfc}\left(\sqrt{\frac{t}{\tau_\eta}}+\frac{z}{2\sqrt{t\eta}}\right)-\exp\left(\frac{t}{\tau_{\mathrm{h}}}\right)\exp\left(\frac{z}{\sqrt{\tau_{\mathrm{h}}\eta}}\right)\operatorname{erfc}\left(\sqrt{\frac{t}{\tau_{\mathrm{h}}}}+\frac{z}{2\sqrt{t\eta}}\right)\right]. \tag{28}$$

Parameters τ_α, τ_η, and τ_{h} have clear physical sense to be the characteristics of a corresponding process

provided that the process dominates. Really, $\tau_\alpha = d^2/(9.7\eta)$ is the characteristic time of the temperature equalization of blood and its surrounding tissue; $\tau_\eta = 1/(\beta^2\eta)$ is the characteristic time of the temperature equalization over tissue depth due to thermal conductivity; and $\tau_h = 1/(h^2\eta)$ is the characteristic time of heat exchange between the tissue surface and environment.

Thus, Eqs. (22)–(28) describe the temperature changes of the two tissue components under irradiation by a light δ-pulse or the Green function of the corresponding problem expressed via the characteristic times of the respective thermal processes. Spatial and temporal temperature distributions of [55,56] are also expressed via characteristic times of axial τ_η and radial τ_w heat spreading to simplify the model of heat transfer through tissues and to generalize the calculations. (As pointed out above, we neglected τ_w or assumed that $\beta w \gg 1$, where w is the spot size at the entrance to the medium.) However, a number of rough assumptions were made in [55,56]. According to [55], $\tau_\eta = 0.7/(\beta^2\eta)$ to be close to the stated above. Eqs. (22)–(28) comprise additionally the characteristic times of other thermal mechanisms, namely τ_α of heat exchange between blood and basic tissue and τ_h of heat losses through the interface. This enables us to vividly present the results and easily analyze them.

We assumed above $U = 0$. Then according to Eqs. (22) and (23), $\overline{T}_1$ and $\overline{T}_2$ tend to be the same as T_3 at $t \to \infty$ to contradict with the reality. It is required to discard the last term of Eqs. (22) and (23) to provide the correct asymptotic solution $\overline{T}_1(t \to \infty) = \overline{T}_2(t \to \infty) = T_{10}$. One could believe that the organism thermal source ($U \neq 0$) would compensate δT in these formulas. It should be noted that δT value plays an essential role only when $\overline{T}_1$ and $\overline{T}_2$ are close to T_{10}, i.e. when the tissue overheating is disappearing. So the working formulas for further quantitative calculations are

$$\Delta T_1(t,z) = \overline{T}_1(t,z) - T_{10} + \frac{A_1 F(t,z)}{2} - A_1 F_1(t,z,\beta)\frac{\sqrt{\tau_\eta}}{\sqrt{\tau_\eta} - \sqrt{\tau_h}}, \tag{29}$$

$$\Delta T_2(t,z) = \overline{T}_2(t,z) - T_{20} + \frac{A_2 F(t,z)}{2} - A_2 F_1(t,z,\beta)\frac{\sqrt{\tau_\eta}}{\sqrt{\tau_\eta} - \sqrt{\tau_h}}, \tag{30}$$

where ΔT_1 are ΔT_2 the temperature excesses over the normal temperature.

Even without computations by Eqs. (29) and (30), one can get the insight into thermal processes in tissues by evaluating the time parameters τ_α, τ_η, and τ_h.

Characteristic time τ_α of the temperature equalization of blood and basic tissue is proportional to d^2. So τ_α increases rapidly with mean capillary diameter. If d changes from 2.5 to 40 μm, the τ_α range is 4.6×10^{-6} to 1.2×10^{-3} s. Parameter $\tau_h \propto 1/h^2$. As noted above, h ranges from 5 to 10^5 m^{-1} depending on the environment and convection kind (free or forced). This corresponds to the range of τ_h from 2.9×10^5 to 7×10^{-4} s. Characteristic time $\tau_\eta \propto 1/\beta^2$, i.e. the more the light absorption, the faster the temperature would equalize over tissue depth due to thermal conductivity. Taking into account possible changes in β given in Table 1, one gets τ_η values of about 0.2–60 s. So the widest range is for τ_h (9 orders of magnitude), its value can be both greater and less than τ_α, and $\tau_\alpha \ll \tau_\eta$ always.

Note that initial Eqs. (18) and (19) do not contain directed perfusion component $\vec{v}_2$. If $v_2 \neq 0$, one should add characteristic time $\tau_v = L/v_2$ (L being the path length along $\vec{v}_2$, where temperature changes due to the perfusion are estimated) to time parameters τ_α, τ_η, and τ_h. When the velocity vector is almost parallel to the tissue surface, L is about the light spot diameter. Let $L = 0.01$ m and $v_2 = 10^{-5}$ m/s, then $\tau_v = 10^3$ s. If the velocity vector is close to the surface normal, the characteristic value of L will be z_0 or some mm depending on λ, and τ_v is about seconds.

7. Particular cases of Eqs. (29) and (30)

I. At $t = 0$

$$\Delta T_{1,2} = A_{10,20}\exp(-\beta z), \tag{31}$$

i.e. the spatial distributions of temperature rises of blood and basic tissue are similar to the distribution of the fluence rate over depth z.

II. At $\tau_h = \infty$

$$\Delta T_{1,2} = A_{1,2} F(t,z). \tag{32}$$

III. At $\tau_h = 0$

$$\Delta T_{1,2} = A_{1,2} F_2(t,z), \tag{33}$$

where

$$F_2(t,z) = \exp\left(\frac{t}{\tau_\eta}\right)\left[\exp(-\beta z)\operatorname{erfc}\left(\sqrt{\frac{t}{\tau_\eta}} - \frac{z}{2\sqrt{t\eta}}\right) - \exp(\beta z)\operatorname{erfc}\left(\sqrt{\frac{t}{\tau_\eta}} + \frac{z}{2\sqrt{t\eta}}\right)\right]. \tag{34}$$

IV. At $\tau_\alpha = \infty$, when components 1 and 2 do not interact with each other,

$$\Delta T_{1,2} = A_{10,20}\left[\frac{F(t,z)}{2} - F_1(t,z,\beta)\frac{\sqrt{\tau_\eta}}{\sqrt{\tau_\eta} - \sqrt{\tau_h}}\right]. \tag{35}$$

V. At $\tau_\alpha = 0$, when the temperatures of components 1 and 2 equalize in a moment and the medium behaves itself as a single-component one,

$$\Delta T_{1,2} = \Delta T_{1,2}\Big|_{\tau_h=\infty} - A_0\left[F_1(t,z,\beta)\frac{\sqrt{\tau_\eta}}{\sqrt{\tau_\eta}-\sqrt{\tau_h}}\right]. \quad (36)$$

VI. At $z = 0$

$$\Delta T_{1,2} = A_{1,2}\left\{\exp\left(-\frac{t}{\tau_\eta}\right)\mathrm{erfc}\left(\sqrt{\frac{t}{\tau_\eta}}\right) - \frac{\sqrt{\tau_\eta}}{\sqrt{\tau_\eta}-\sqrt{\tau_h}}\left[\exp\left(\frac{t}{\tau_\eta}\right)\mathrm{erfc}\left(\sqrt{\frac{t}{\tau_\eta}}\right) - \exp\left(\frac{t}{\tau_h}\right)\mathrm{erfc}\left(\sqrt{\frac{t}{\tau_h}}\right)\right]\right\} \quad (37)$$

For small time moments, Eq. (37) is

$$\Delta T_{1,2} = A_{1,2}\left[1 - \frac{2\sqrt{t}(\sqrt{\tau_\eta}+\sqrt{\tau_h})}{\sqrt{\tau_\eta\tau_h}}\right]. \quad (38)$$

Maximal overheating $\Delta T_2^{\max}$ of blood occurs at the first moments after the irradiation by a δ-pulse. Let a pulse laser create E_0 of about 2×10^3 J/m^2 at $z = 0$. The values of $\Delta T_2^{\max}$ and $\Delta T_2^{*\max}$ calculated by Eq. (31) for this energy density and various wavelengths with and without accounting for the "sieve" effect, respectively, are given in the two last columns of Table 2 for tissue in air and $\varphi = 0$, $d = 40$ μm. The results show that blood is highly overheated at the first moments after the irradiation. This is especially noticeable in the short-wavelength spectral range, where the temperature excess over the normal temperature can achieve several dozens of degrees. Note that the said maximal temperature values are computed at the above energy surface density selected for the estimations. At the same time, $\Delta T_2^{\max}$ and $\Delta T_2^{*\max}$ are proportional to E_0, i.e. the larger E_0 value, the higher the maximal temperatures. Had the said overheating lasted for a long time, this would lead to irreversible destructive changes in tissues. However, it

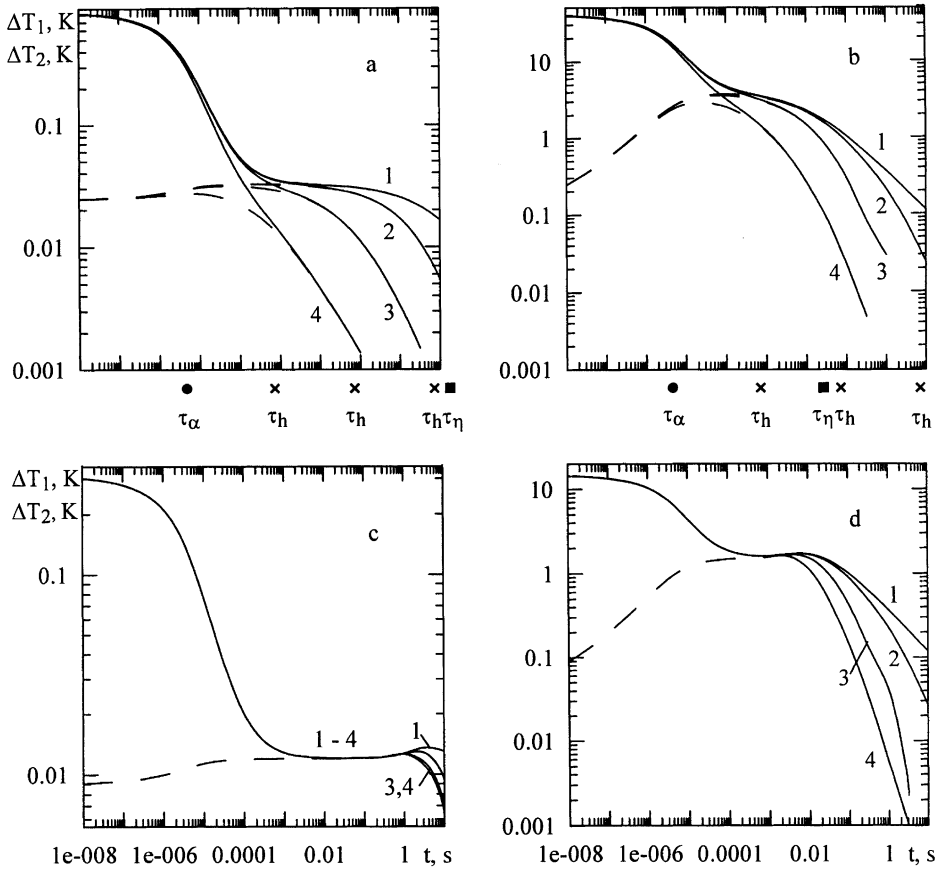

Fig. 1. Temperature excess of basic tissue ΔT_1 (dashed curves) and blood ΔT_2 (solid) over normal temperature as a function of time after irradiation for weak (a and c, $\lambda = 600$ nm, $C_V = 0.01$) and strong (b and d, $\lambda = 400$ nm, $C_V = 0.1$) absorption at $z = 0$ (a,b) and $z = z_0 = 1.57$ (c) and 0.17 mm (d), $\tau_h = \infty$ (curves 1), 7.1 (2), 0.071 (3), and 0.00071 s (4). On the abscissa axis, characteristic times τ_α (●), τ_h (×), and τ_η (■) are shown.

will be shown below that the duration of this process is very small to do not, probably, cause the irreversible transformations of the organic matter.

8. Features in thermal processes

All the features considered below are obtained at $d = 2.5$ μm to correspond to $\tau_\alpha = 4.6 \times 10^{-6}$ s as well as at $r_1 = 0.02$ and $E_0 = 2 \times 10^3$ J/m^2. Due to small d, the "sieve" effect was neglected. The calculations are performed by Eqs. (29) and (30) with accounting for Eq. (25). Although the formulas do not contain τ_α explicitly, we note its values in view of the approximation of Eq. (25) by Eq. (24).

First, turn our attention to the influence of τ_h on the temperature kinetics at $\lambda = 600$ nm, $C_V = 0.01$ and $\lambda = 400$ nm, $C_V = 0.1$ (Fig. 1). The data are given for $z = 0$ and z_0. In the case of the weak and strong absorption, τ_η values are 17.5 and 0.026 s, respectively. Fig. 1 shows that even for the weak absorption the blood temperature rise at the surface can be about 1 K. On the whole, the temperature effects are small in this case to be some fractions of a degree or less. At the strong absorption, the blood temperature can initially achieve several degree dozens. Although $\tau_\alpha = 4.6 \times 10^{-6}$ s, the complete equalization of the blood and basic tissue temperatures is longer than τ_α by the two orders of magnitude. This was qualitatively pointed out above. With that one can see the enhanced surface temperature by several degrees up to fractions of a second. At $t < 10^{-5}$ s, the temperature transformations occur only due to the heat exchange between blood and basic tissue. With that $\overline{T}_2$ decreases, but $\overline{T}_1$ rises. These processes are described by coefficients A_2 and A_1 in Eqs. (30) and (29). Further, temperatures $\overline{T}_1$ and $\overline{T}_2$ are equalized to operate the heat transfer to the external medium. The smaller τ_h, the more rapidly the surface gets cold. The role of τ_h at some depths becomes more efficient at larger times than at the surface. Starting from $t > 0.01$ s, the thermal conductivity mechanism is operated, and when $\tau_h = \infty$, the temperature drop of tissue is due to only heat spreading over the medium depth.

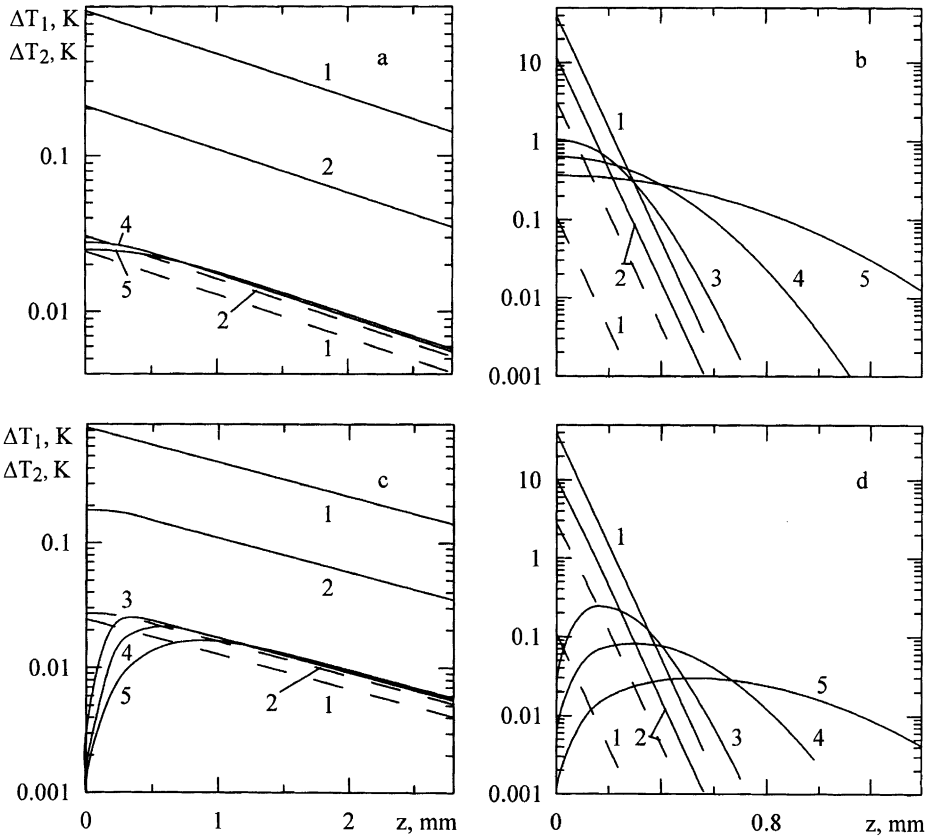

Fig. 2. Temperature excess of basic tissue ΔT_1 (dashed curves) and blood ΔT_2 (solid) over normal temperature as a function of depth z for weak (a and c, $\lambda = 600$ nm, $C_V = 0.01$) and strong (b and d, $\lambda = 400$ nm, $C_V = 0.1$) absorption at $t = 0$ (curves 1), 10^{-5} (2), 0.1 (3), 0.32 (4), and 1 s (5); $\tau_h = 4.5 \times 10^3$ (a,b) and 7.1×10^{-4} s (c,d).

Fig. 2 illustrates $\Delta T_{1,2}$ as a function of z at different τ_h. Value $\tau_h = 4.5 \times 10^3$ s is typical for tissue contacting with air. This case essentially corresponds to $\tau_h = \infty$ to be described by Eq. (23). Value $\tau_h = 7.1 \times 10^{-4}$ s corresponds to power water flow near tissue, and the temperature kinetics is close to Eq. (33). One can see from Fig. 2 that at $t \ll 7.1 \times 10^{-4}$ s, when the kinetics is not defined by τ_h, the temperature dependencies are exponential, what follows from Eqs. (29) and (30) that give

$$\Delta T_{1,2} = A_{10,20} \exp(-\beta z) \exp(t/\tau_\eta). \quad (39)$$

The differences in the curves are related with heat exchange between blood and basic tissue. The exponential behavior is violated with increasing t stating from small z. This is owing to the heat transfer action providing the heat outflow inside the layer. When the outflow rate via the surface is very high, so that $\tau_h \ll \tau_\eta$, the thermal conductivity mechanism fails to hold the heat in the subsurface layer, which gets colder than deeper layers in some time. With higher absorptivity and lower τ_h, thermal conductivity assists to more intense heat transfer inside the medium. At larger depths, the more the time, the higher the temperature. This follows also from Eqs. (29) and (30), which are reduced to Eq. (39) at $z \to \infty$ independent of the parameters of the medium.

Thus, the proposed model for heating biological tissues by a short light pulse enables the thermal fields to be analytically evaluated with regarding all the parameters of the problem. The representation of the solutions via characteristic times of thermal processes permits one to estimate essential and inessential factors, to reduce a number of parameters that should be regarded, and to thereby simplify the final results. This model will be a basis for studying temperature distributions over biological tissues irradiated by a pulse of finite duration as applied to various medical and biophysical applications.

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